THERMAL AGITATION OF ELECTRICITY IN CONDUCTORS

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ABSTRACT

Statistical fluctuation of electric charge exists in all conductors, producing random variation of potential between the ends of the conductor. The effect of these fluctuations has been measured by a vacuum tube amplifier and thermocouple, and can be expressed by the formula $\bar{I}^2=(2kT/\pi)\int_0^\infty R(\omega)|Y(\omega)|^2 d\omega$. I is the observed current in the thermocouple, k is Boltzmann's gas constant, T is the absolute temperature of the conductor, $R(\omega)$ is the *real* component of impedance of the conductor, $Y(\omega)$ is the transfer impedance of the amplifier, and $\omega/2\pi=f$ represents frequency. *The value of Boltzmann's constant* obtained from the measurements lie near the accepted value of this constant. *The technical aspects of the disturbance* are discussed. In an amplifier having a range of 5000 cycles and the input resistance R the power equivalent of the effect is $\bar{V}^2/R=0.8\times10^{-16}$ watt, with corresponding power for other ranges of frequency. The least contribution of *tube noise* is equivalent to that of a resistance $R_c=1.5\times10^5 i_p/\mu$, where i_p is the space current in milliamperes and μ is the effective amplification of the tube.

IN TWO short notes[1] a phenomenon has been described which is the result of spontaneous motion of the electricity in a conducting body. The electric charges in a conductor are found to be in a state of thermal agitation, in thermodynamic equilibrium with the heat motion of the atoms of the conductor. The manifestation of the phenomenon is a fluctuation of potential difference between the terminals of the conductor which can be measured by suitable instruments.

The effect is one of the causes of that disturbance which is called "tube noise" in vacuum tube amplifiers.[2] Indeed, it is often by far the larger part of the "noise" of a good amplifier. When such an amplifier terminates in a telephone receiver, and has a high resistance connected between the grid and filament of the first tube on the input side, the effect is perceived as a steady rustling noise in the receiver, like that produced by the small-shot (Schrot) effect under similar circumstances. The use of a thermocouple or rectifier in place of the telephone receiver allows reasonably accurate measurements to be made on the effective amplitude of the disturbance.

It had been known for some time among amplifier technicians that the "noise" increases as the input resistance is made larger. A closer study of this phenomenon revealed the fact that a part of the noise depends on the resistance alone and not on the vacuum tube to which it is connected. The true nature of the effect being then suspected, the temperature of the re-

[1] Johnson, Nature **119**, p. 50, Jan. 8, 1927; Phys. Rev. **29**, p. 367 (Feb. 1927).

[2] The possibility that under certain conditions the heat motion of electricity could create a measurable disturbance in amplifiers has been recognized on theoretical grounds by W. Schottky (Ann. d. Phys. **57**, 541 (1918)). Schottky considered the special case of a resonant circuit connected to the input of a vacuum tube, and concluded that there the effect would be so small as to be masked by the small-shot effect in the tube.

sistance was varied and the result of that test left little doubt that the thermal agitation of electricity in the resistance element was being observed. Further experiments led to the expression of the effect by a formula which, except for a small difference in the numerical constant, was the same as that later developed by Dr. H. Nyquist[3] on a wholly theoretical basis.

The chief results of the measurements may be summarized as follows. The mean-square potential fluctuation over the conductor is proportional to the electrical resistance and the absolute temperature of the conductor. It is independent of the size, shape or material of the conductor. Its apparent magnitude depends on the electrical characteristics of the measuring system as well as on those of the conductor itself. The quantitative data yield a value for Boltzmann's gas constant which agrees well with that obtained by other methods. It is the purpose of this article to describe in more detail these results and the methods by which they were derived, and to discuss the limit which the phenomenon imposes on amplification by vacuum tubes.

Experimental Method and Apparatus

The significance of the mathematical expression for the effect will be developed with the aid of the generalized circuit diagram of Fig. 1. Z is the conductor under investigation, A the amplifier to which it is connected, J the thermocouple ammeter. The amplifier A is characterized by a complex

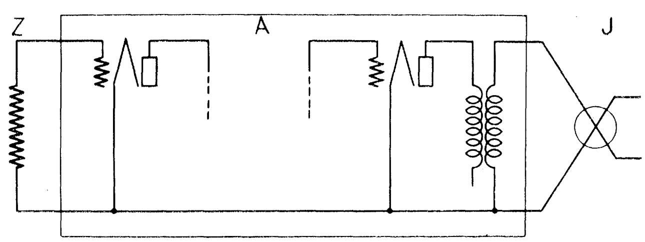

Fig. 1. Simplified diagram of the circuit.

transfer admittance $Y(\omega)$, defined as the ratio of output current to applied input voltage at any frequency $\omega/2\pi = f$. The complex impedance of the element Z has a real resistance component $R(\omega)$ which is also a function of the frequency. The random fluctuation of potential across the input element, arising from the thermal motion of its electric charges, should give rise to a mean-square current $\overline{I^2}$ in the thermocouple according to the equation[4]

$$\overline{I}^2 = (2kT/\pi) \int_0^\infty R(\omega) \mid Y(\omega) \mid ^2 d\omega. \tag{1}$$

T is here the absolute temperature of the input element and k is Boltzmann's gas constant.

[3] Nyquist, Phys. Rev. **29,** 614 (1927); **32,** 110 (1928).
[4] Nyquist, Phys. Rev. **32,** 110 (1928), Eq. (6).

The value of the integral in this expression may be found by graphic ntegration of the curve formed by the experimentally determined values of $R(\omega)|Y(\omega)|^2$ plotted against ω. In most of the present work the input element Z was a high resistance in parallel with its own shunt capacity and that of its leads and of the input of the amplifier. In such a combination the real resistance component $R(\omega)$ is related to the pure resistance R_0 and the capacity C according to

$$R(\omega)=R_0/(1+\omega^2C^2R_0^2) \tag{2}$$

Two cases now arise. If the input element and the amplifier are so chosen that $R(\omega)$ does not change much over the frequency range of the amplifier, then it is permissible to use the mean value of $R(\omega)$, obtained to a sufficient degree of approximation by replacing ω by ω_0, the frequency of maximum amplification. The factor $R(\omega)$ can then be placed outside the sign of integration and Eq. (1) becomes

$$\bar{I}^2=\frac{2kTR_0}{\pi(1+\omega_0{}^2C^2R_0{}^2)}\int_0^\infty |Y(\omega)|^2 d\omega. \tag{3}$$

If, on the other hand, $R(\omega)$ cannot be considered constant over the frequency range then the integral must be used as it stands in Eq. (1). The method of determining the various quantities involved in these expressions 1, 2 and 3 will be taken up as the apparatus is described in greater detail.

The amplifier which was used consisted of six stages of audion tubes, suitably coupled, and chosen according to the voltage and power requirements of their various positions. The coupling between tubes was done, with the exception of one interstage, either by transformers or by choke coils and condensers. The exception was a coupling consisting of either a resonant circuit, as shown in Fig. 2, or a band-pass filter, used for the purpose of limiting the amplification to a selected band of frequencies. A comparatively

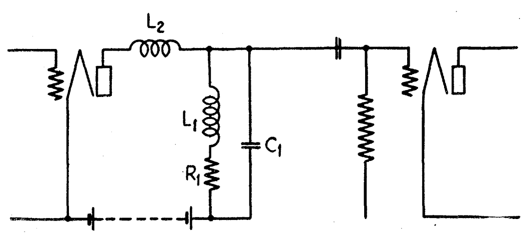

Fig. 2. Diagram of the resonant coupling.

narrow frequency band was admitted by the resonant circuit, the width of the band depending on the relative magnitude chosen for the various components of this system. The band-pass filter, on the other hand, had the frequency range of 500 to 1000 periods per second, with a much sharper cut-off at the limits than the simple resonant circuit.

The amplifier was enclosed in a steel cabinet. It was elaborately shielded against electric, magnetic, acoustic and mechanical shocks, but it was not always entirely free from these disturbances. Regeneration was apparently negligibly small.

The last tube of the amplifier was coupled by a transformer to a vacuum thermocouple, used with a microammeter as the indicating instrument. The couple was calibrated against a direct current meter of established accuracy. Conveniently, the deflection of the microammeter was closely proportional to the square of the current in the thermocouple.

For the calibration of the amplifier, current sufficiently free from harmonics was obtained from a vacuum tube oscillator. The current was measured by a thermocouple similar to that used in the output. An attenuator of the type described by A. G. Jensen[5] was used so that a known fraction of the current was passed through a resistance of one ohm connected across the input of the amplifier, producing a known input voltage. The corresponding output current was observed. Here, however, a certain correction had

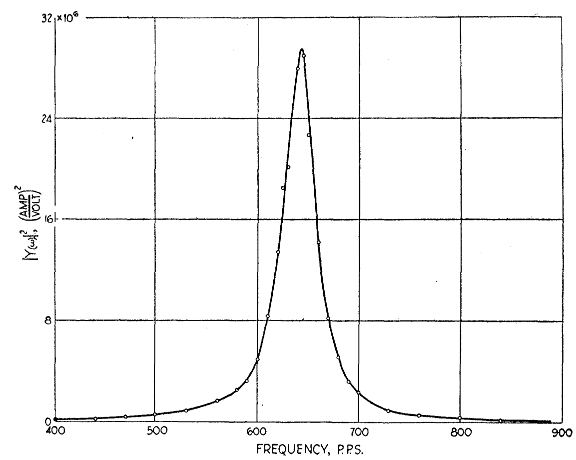

Fig. 3. Typical resonance curve.

to be applied. The amplifier itself, and chiefly the first vacuum tube, produced a disturbance which caused the output meter to deflect even without any external source of input voltage. This "zero deflection" was observed frequently and was subtracted from the total deflection. This could be done, since the mean-square currents added linearly in the deflection, and the value of the mean-square current was desired. The quantity $|Y(\omega)|^2$ was then given by the ratio of the mean-square output current to the mean-square input voltage, which in turn was the ratio of the corrected output deflection to the input deflection, times a multiplier derived from the attenuator setting and the calibrations of the couples. Depending on whether absolute or only comparative measurements were to be made, the determination of $|Y(\omega)|^2$ was done at a series of frequencies or only at the single frequency of maximum amplification. In the latter case the factor $|Y(\omega)|^2$ was only a sort of amplifi-

[5] Jensen, Phys. Rev. **26**, 118 (1925).

cation factor which served as a check on the constancy of the amplifier, while in the former case it was used in the determination of the integral of Eqs. (1) and (3).

A typical curve of amplification factor versus frequency is shown in Fig. 3, obtained from the data for one particular condition of the resonant circuit. The area under curves of this type, establishing the value of the integral, was measured by a planimeter. Since, however, the curves could not be plotted for the full range of the integrals, from zero to infinite frequency, it was necessary to estimate that part of the total area which was limited by the extremes of the characteristic curves. Some of the curves were actually carried out so far (that of Fig. 3, for instance, from 200 to 2200 periods per second) that the further extension to infinite and to zero frequency would have added an almost certainly negligible amount to the total area. Other curves extended only far enough to delimit the greater part of the area, and to the area under these was added a correction arrived at by comparison with the more extensive curves.

An alternating current bridge was used for measuring the resistances R_0 and the capacities C at the frequencies involved. The capacities were of the order 50 mmf. for the input to the amplifier, 10 mmf. for the resistance and from 5 to 150 mmf. for the leads. The effective resistance of the input (grid to filament of the first amplifier tube) was of the order 15 megohms. The measurements of input resistance and capacity were made while the resistance unit was connected in the input of the amplifier, and the amplifier was in the normal operating condition.

Temperature baths were prepared in Dewar flasks when it was desired to keep the resistances at a temperature other than that of the room. The heating or cooling agents were boiling water, melting ice, solid carbon dioxide in acetone, and old liquid air. The temperatures were measured by a platinum resistance thermometer.

There remain to be described the resistance units upon which the experiments were done. These were chosen so as to include materials of different properties, such as high or low resistivity, positive or negative temperature coefficient, metallic or electrolytic conduction, light ions or heavy ions. In value the resistances ranged from a few thousand ohms to a few megohms.

For values of more than one-half megohm commercial "grid leak" resistances were used, made with India ink on paper. Another type of commercial product, made of carbon filament wound on lavite, covered the range of resistance below one megohm. Platinum and copper resistances were used in the form of thin films deposited on glass by evaporation. A resistance of nearly a half megohm was made of Advance wire, wound non-inductively. This was provided with a tap so that one-third or two-thirds of it could be used.

Electrolytic resistances were made of aqueous solutions of the salts NaCl, $CuSO_4$, K_2CrO_4, $Ca(NO_3)_2$, and of a solution of sulphuric acid in ethyl alcohol. Glass tubes about 15 cm long, some of capillary size and some larger, were filled with the different solutions of such strength as to give them all about the same resistance, and this was repeated with another resistance value.

Measurements and Results

A considerable part of the work consisted of comparative measurements in which the characteristics of the amplifier did not need to be known. In these circumstances only the maximum amplification was determined. It was convenient in such cases to think of the resistance as impressing on the amplifier a mean-square potential $\overline{V^2}$. By this method of comparison was determined the fact that the phenomenon is independent of the material and shape of the resistance unit and of the mechanism of the conduction,[6] but does depend on the electrical resistance. A few of the results are reproduced in Fig. 4. They are expressed in terms of $\overline{V^2}$, the apparent mean-square potential fluctuation, plotted against the resistance component $R(\omega)$. The points lie close to a straight line. The quantity $W=\overline{V^2}/R(\omega)$, which may be

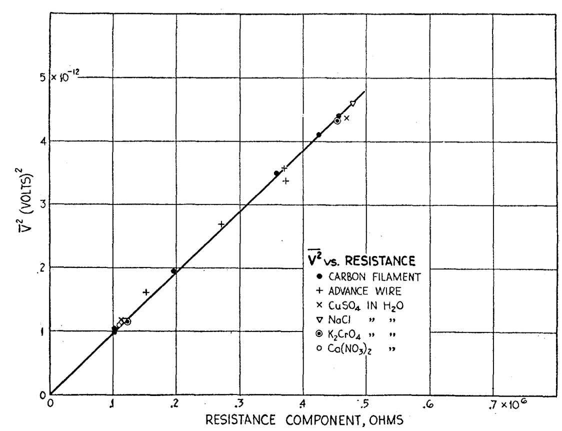

Fig. 4. Voltage-squared vs. resistance component for various kinds of conductors.

called the power equivalent of the effect, is independent of all the variables so far considered, including the electrical resistance itself.

The effect of the shunt capacity C across the conductor is shown in Fig. 5. In this case the abscissae are the measured values of resistance R_0, the circles marking the observed values of the apparent $\overline{V^2}$. These values of $\overline{V^2}$ reach a maximum and then actually decrease as the resistance is indefinitely increased. Obtaining a factor of proportionality K from the initial slope of the curve and using the measured value of C and ω for the calculation of $\overline{V^2}=KR(\omega)=KR_0/(1+\omega^2C^2R_0^2)$, the expected values of $\overline{V^2}$ were calculated. These are represented by the curve in Fig. 5. The course of the calculated curve agrees well with that for the observed points. The agreement was also verified by using a fixed resistance and adding known shunt capacities up to as high as 60,000 mmf.

[6] Resistances such as thermionic tubes and photoelectric cells, perhaps all resistances not obeying Ohm's law, are exceptions to this rule. In these the statistical conditions are different.

The effect of the temperature of the resistance element was studied chiefly by the same comparative method that was used for the varied resistances. The experiments were done on the Advance wire resistance and on carbon filament resistances over the temperature range from $-180°C$ (liquid air) to $100°C$ (boiling water). They were also done on two liquid resistances made of alcohol and sulphuric acid, covering a range of tempera-

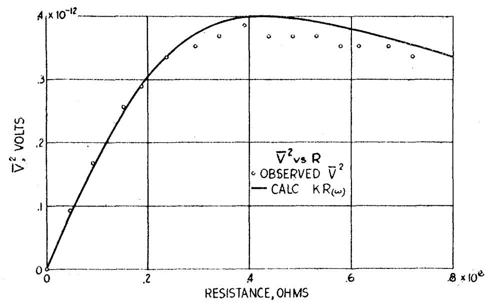

Fig. 5. Voltage-squared vs. resistance with fixed shunt capacity; frequency 635 p.p.s., capacity 577 mmf.

tures from $-72°C$ to $90°C$. The resistance values used in the computations were those measured at the various temperatures. Advance wire and carbon filaments changed very little in resistance over the temperature range used for them, while that of the liquid elements changed tremendously, increasing for lower temperatures. In all cases, however, the virtual power $\overline{V}^2/R(\omega)$

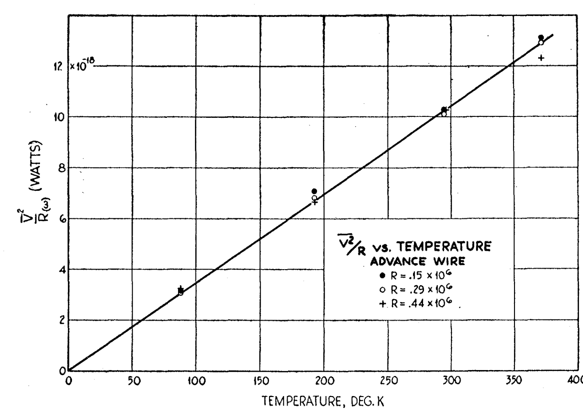

Fig. 6. Apparent power vs. temperature, for Advance wire resistances.

was proportional to the absolute temperature of the resistance element. Fig. 6 shows graphically the results for the three Advance wire resistances. The other resistances gave values falling closely on the same straight line as these.

There is finally to be recounted the verification of Eqs. (1) and (3) as they involve the electrical properties of the measuring system. The method of obtaining the frequency characteristic of the amplifier, the graphic integration of this curve and the corrections that were applied, have already been described. For these determinations the amplifier was altered in various ways, but usually by changing the resonant circuit forming one of the interstage couplings, or by replacing this circuit by the band-pass filter. The natural frequency of the resonant circuit was changed, by means of the inductance or condenser, over the range of 300 to 2000 periods per second. The sharpness of resonance was varied by changing either the resistance in the resonant circuit or the inductance in series with this circuit and the tube. The area under the characteristic curve could thus be made larger or smaller over a considerable range.

The input resistance element, in all of this work, was kept at only slightly above room temperature. It was of the "grid leak" type for most of those measurements in which the resonant circuit was employed, while the Advance

TABLE I. *Determination of Boltzmann's Constant.*

No.	f	T	$R(\omega)$	$\int_0^\infty \lvert Y(\omega)\rvert^2 d\omega$	$\int_0^\infty \lvert Y(\omega)\rvert^2 R(\omega) d\omega$	ΔS	$\overline{I^2}$	k
	p.p.s.	°K	$\times 10^{-6}$	$\times 10^{-10}$	$\times 10^{-16}$	%	$\times 10^{6}$	$\times 10^{16}$
1	1010	298	.526	.213		12.0	2.7	1.27
2	2023	"	.470	.272		26.2	2.8	1.15
3	1418	"	.508	.361		15.6	3.8	1.09
4	"	"	"	.188		42.3	1.8	.99
5	295	"	.548	.252		3.3	3.1	1.18
6	"	"	"	.202		15.8	2.0	.95
7	302	"	"	.221		12.4	2.7	1.18
8	"	"	"	.195		15.3	2.3	1.13
9	653	"	.541	.747		6.8	10.4	1.14
10	"	"	"	.645		12.9	8.6	1.30
11	1418	"	.508	.286		18.5	3.5	1.26
12	"	"	"	.161		41.3	1.7	1.09
13	1465	"	.505	1.93		18.7	21.2	1.14
14	"	"	"	1.75		20.3	19.1	1.14
15	635	"	.541	.594		8.8	8.9	1.46
16	"	"	"	.139		35.0	2.1	1.47
17	"	"	"	.597		10.9	7.8	1.28
18*	643	295	.44 ±		.439	0	11.0	1.38
19	645	297	"		.396	0	11.1	1.49
20	1830	301	"		.913	0	19.8	1.13
21	500–1000	299	"		.831	0	25.9	1.64
22	"	300	"		.662	0	21.5	1.70
23	"	300	"		.832	0	26.0	1.63

* The resonance curve for this determination is that reproduced in Fig. 3.

wire resistance was used as input element in connection with the band-pass filter. The resistance and capacity of each was measured accurately *in situ*, the resistance being of the order of one-half megohm.

The results will be presented in terms of the value of Boltzmann's constant k, as calculated from the data according to Eqs. (1) or (3). The last column of Table I contains these calculated values, while in the other columns of the table are indicated some of the experimental conditions involved in each determination. The columns of the table indicate, in order, the resonance frequency or frequency range of the selective circuit, the temperature and the ohmic value of the input resistance component, and the value of one or the other infinite integral. The column under ΔS contains the part, in percent, which the estimated area contributes to the total integral. $\bar{I}^2$ is the current-squared observed in the thermocouple.

The average of the values of k in the last column of the table is 1.27×10^{-16} ergs per degree, with a mean deviation of 13 percent. The average is 7.5 percent lower than the accepted value[7] of k, 1.372×10^{-16} ergs per degree. The series of measurements, therefore, yields a value of k which is correct within the mean deviation. I believe, however, that the method is capable of a much higher accuracy than that obtained here. This leads to a discussion of the possible sources of error.

An inspection of the tabulated data reveals no systematic relation of the value of k to the numbers in any one column, except that the results of the last three measurements, made with the band-pass filter, are higher than the rest. The deviations are apparently distributed at random. Among the quantities which enter Eq. (1) there can be little question of the accuracy of the temperature T and the resistance component $R(\omega)$. The error is therefore to be sought in the current-squared, $\bar{I}^2$, and in the integral of $|Y(\omega)|^2 d\omega$. The latter is indeed open to the criticism that the correction term for the area was often large and therefore uncertain, but this cannot be held against the last six determinations. It is possible, because of feedback through the internal capacities of the tubes, that the amplification of the system was not the same when the resistance alone was connected across the input as when this was shunted by the low resistance of the calibration circuit. It is not clear, however, how such a change in amplification could make the values of k sometimes too large and at other times too small. The other debatable factor, $\bar{I}^2$, may be questioned in two respects. There was a "zero reading," due to tube noise, amounting usually to about ten percent of the total reading of the meter used for measuring $\bar{I}^2$. This zero reading, however, was always quite constant. The total reading, on the other hand, was not constant but fluctuated over a range comprising about five percent of the total reading. This unsteadiness might be caused by disturbances generated either within the amplifier or coming from the outside, or it might represent a natural fluctuation in the phenomenon itself. Two different procedures were used in obtaining the reading. At first a single datum was used, taken after watching the needle for perhaps thirty seconds. Judgment placed this value nearer the lower limit of the excursion of the needle, since disturbances would tend to make the readings high. For each of the last six sets of measurements, Nos. 18 to 23, the average of a series of readings was used. The meter was read at about one hundred equally spaced intervals of time, before and after obtaining the frequency characteristic of the system. The average of these readings was slightly greater than that arrived at by judging the undisturbed position of the needle.

[7] Int. Crit. Tables, v. 1, p. 18.

Whatever the cause of the deviation, the fact remains that there is essential agreement in all respects between the theory and the experimental results. It is remarkable that the same apparatus by which it is possible to determine the charge on the electron by means of the small-shot effect, can by a slight change in procedure be made to yield an independent measurement of Boltzmann's gas constant.

Technical Aspects of the Phenomenon

Since the thermal agitation places a limit on what can be done by amplifiers, it will be of interest to discuss this limit in terms more commonly used than those of Eq. (1). It is more convenient to speak of a disturbance to an amplifier as a voltage fluctuation at the input than as a current fluctuation at the output which has been used here. A voltage fluctuation across the input of the amplifier cannot, in general, logically be derived from Eq. (1). Certain simplifying conditions may be assumed, however, which make such a derivation plausible. Let us take a circuit having constant amplification over the frequency range f_1 to f_2, zero amplification outside this range. The input resistance component may be assumed to have the constant value R within the frequency range of the circuit. Eq. (1) may then be written

$$\bar{I}^2 = (2kTRY^2/\pi)(\omega_2-\omega_1) = 4kTRY^2(f_2-f_1) = 4kTR(f_2-f_1)I^2/V^2, \qquad (4)$$

where V is the r.m.s. voltage, having any frequency within the pertinent range, applied at the input of the amplifier, and I is the current produced at the output of the circuit. Now the voltage V may be given such a magnitude that the corresponding output current-squared equals that produced by the thermal agitation in the resistance R. The value of V^2 can then be considered equivalent to the voltage-squared generated by the thermal agitation and may be denoted by $\overline{V}^2$. Eq. (4) becomes

$$I^2 = 4kTR(f_2-f_1)\overline{I^2}/\overline{V}^2\ ;$$

$$\overline{V^2} = 4kTR(f_2-f_1) = WR\ ; \qquad (5)$$

$$W = \overline{V^2}/R = 4kT(f_2-f_1). \qquad (6)$$

W is the virtual power, which in its effect on the output meter is equivalent to an actual power of this value dissipated in the input resistance. For an amplifier operated at room temperature and covering the approximate voice frequency range of 5000 cycles this power is

$$W = 4\times 1.37\times 10^{-16}\times 300\times 5000\times 10^{-7} = 0.82\times 10^{-16}\ \text{watt.}^{8}$$

If the input resistance is one-half megohm the mean-square voltage fluctuation is $\overline{V}^2 = WR = 0.82\times 10^{-16}\times 0.5\times 10^6 = 0.41\times 10^{-10}$ (volts)^2, to which corresponds an apparent input voltage of 6.4 microvolts. A value very close

[8] The value of 10^{-18} watt given in the cited notes was for the narrower frequency band of a resonant circuit.

to this was actually observed when the experimental amplifier was given approximately the characteristics assumed here.

For input elements other than pure resistance the problem cannot be handled in this simple way, but Eq. (1) should still apply as it has been seen to do in the case of the resistance-capacity combination. When a resonant circuit was used in the input, however, the observed value of $\bar{I}^2$ was somewhat greater than that computed from the characteristics of the system. The difficulty was, no doubt, that it was impossible by means conveniently at hand to shield the inductance coil well enough against magnetic disturbances of external origin.

Towards the problem of reducing the noise in amplifiers caused by thermal agitation the theory makes three suggestions. The first is the use of a low input resistance. This factor, however, is not usually entirely at our disposal, being influenced by the apparatus which supplies the small voltage that is to be amplified. Secondly, the input resistance may be kept at a low temperature. This expedient, too, has practical limitations since the elements which make up the input resistance cannot always conveniently be confined in a small space. The third possible way to reduce the noise consists in making the frequency range of the system no greater than is essential for the proper transmission of the applied input voltage.[9] For the purpose of detecting or measuring a voltage of constant frequency and amplitude one may go to extremes in making the system selective and thereby proportionately reducing the noise. One may, for instance, make use of the great sharpness of tuning obtained by means of mechanical rather than electrical resonance. When, however, the applied voltage varies in frequency or amplitude, the system must have a frequency range large enough to take care of these variations, and the presence of a certain level of noise must be accepted. Beyond this, the chief value of the knowledge gained here lies in the ability it gives to predict the lower limit of noise in any case, so that the impossible will not be expected of an amplifier system.

The noise of thermal agitation is usually the predominant source of disturbance in a well constructed amplifier. Thermionic tubes, however, produce a noise of the same nature, which may exceed the thermal noise when the input resistance component is small. This tube noise is a result of fluctuations in the space current of the tube.[10] When tubes are operated at too low a filament temperature the greater part of these fluctuations are caused by the phenomena named by Schottky "small-shot effect" and "flicker effect." In tubes operated with full space charge limitation of the current, the small-shot effect and flicker effect are largely or perhaps entirely suppressed by the space charge. Under these conditions, and with the grid connected directly to the filament, there is still a remanent fluctuation of current in the tube. In a rough way the amount of this disturbance may be de-

[9] The principle holds for disturbances of other kinds, cf. J. R. Carson, Bell System Tech. J. **4,** 265 (1925).

[10] Johnson, Phys. Rev. **26,** 71 (1925); A. W. Hull, Phys. Rev. **25,** 147 (1925); Phys. Rev. **27,** 439 (1926); W. Schottky, Phys. Rev. **28,** 74 (1926).

scribed in terms of the resistance R_c which when connected between the grid and filament of the tube would cause an equal disturbance in the amplifier, so that the total noise is given by the relation

$$\overline{V^2}=W(R+R_c). \tag{7}$$

The order of magnitude of this resistance R_c, for a tube connected for operation in the range of voice frequencies, may be estimated from the working formula

$$R_c=1.5\times10^5 i_p/\mu, \tag{8}$$

the symbol μ standing for the effective amplification of the tube in combination with its external output impedance, i_p for the space current in the tube measured in milliamperes. Consider, as an example of this rule, a tube having a space current $i_p=0.5$ milliampere and an amplification factor $\mu_0=30$, working into an external impedance equal to its own internal plate resistance. The effective amplification is then $\mu=15$, and the minimum noise of the tube should be equal to that of a resistance $R_c=5{,}000$ ohms in the grid circuit of the tube. If the tube works into an amplifier such as was previously considered this tube noise is therefore equivalent to an effective voltage of .9 microvolt impressed on the grid of the tube. It is not suggested that the predicted minimum noise can be attained with every tube. Many tubes, defective in some way or other, have noise levels much higher than the minimum. It is seen, however, that with the best tubes, properly operated, the tube noise is important only when the resistance component of the input circuit is smaller than the equivalent resistance R_c. With resistances greater than this the thermal agitation should contribute the preponderant part of the noise.[11]

It is interesting to note that in another field of measurements the effect of the thermal agitation of charge in conductors has made itself felt. A group of workers[12] in the Netherlands observed that the deflection of a highly sensitive string galvanometer executed random deviations from the zero position. They ascribed this phenomenon, evidently correctly, in part to "Brownian motion of current" in the galvanometer system. Since then the phenomenon has been further investigated,[13] both theoretically and experimentally, with the conclusion that with a galvanometer the measurement of

[11] In my earlier paper (l.c. 10) the relation of noise to amplification for a number of tubes is shown in Fig. 13. When these data were obtained *a resistance of one-half megohm was connected between the grid and filament of the tube*, a connection which was thought more normal than having no resistance in the grid circuit. The lower limit of noise appears therefore to have been "resistance noise" rather than "tube noise" as this term is now used. Since the noise was measured at the output of the amplifier it is natural that the observed noise should have increased with increasing amplification constant of the experimental tube.

[12] W. Einthoven, F. W. Einthoven, W. van der Holst & H. Hirschfeld, Physica **5**, 358 (1925).

[13] J. Tinbergen, Physica **5**, 361 (1925); G. Ising, Phil. Mag. **1**, 827 (1926); F. Zernike, Zeits. f. Physik **40**, 628 (1926); A. V. Hill, J. Sc. Instr. **4**, 72 (1926).

direct current of less than 10^{-12} ampere becomes unreliable, just as the alternating potential of 10^{-6} volt marks a critical region for amplifiers.

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