The Development of a Practical, Drift Free, Johnson-Noise Thermometer for Industrial Applications

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Abstract

Johnson noise thermometers measure a phenomenon that is directly linked to thermodynamic temperature by a fundamental physical law. The measurement of Johnson noise therefore offers the prospect of realizing a drift free thermometer. Despite previous attempts to produce a practical Johnson noise thermometer for industrial applications, the technique is currently used only in niche research applications to explore discrepancies between practical temperature scales and thermodynamic temperature, or to determine Boltzmann's constant. This has largely been due to the historical use of switched correlators to measure Johnson noise, which limits the sense resistance and measurement bandwidth that can be employed. This constraint limits the Johnson noise signal to levels near the limits of measurement. A new technique that eliminates switching and thereby allows the use of much higher sense resistances and bandwidths to increase the Johnson noise signal is presented. The signal power achieved is significantly higher than for systems using a switched correlator. Results so far indicate that measurement performance is compatible with the requirements of industrial applications. Specifically, uncertainties of < 0.3 ˚C (95 % confidence) were demonstrated for measurements near ambient temperature with a measurement time of only 7 s.

1. Introduction

With commonly-used industrial thermometer types such as thermocouples or platinum resistance thermometers (PRTs), a temperature dependent property is measured that is linked to a temperature scale by calibration. Unfortunately, factors other than temperature (such as contamination of the thermometer materials) can affect the measured property, leading to drift in the indicated temperature. In contrast, Johnson noise thermometers measure a phenomenon that is linked directly to thermodynamic temperature by a fundamental physical law and so do not drift. Any drift in the electronics used to measure the Johnson noise power would lead to drift, but modern electronics are capable of making adequately stable measurements over years or decades. In the most advanced systems employing quantum voltage standards [1] the effect of drift in the measurement electronics is eliminated.

Measuring the extremely small Johnson noise signals with adequate precision and sufficient immunity to extraneous noise sources is at the limits of measurement. Over the years, numerous attempts have been made to produce working Johnson noise thermometers [2]. The principal application in recent years has been exploring discrepancies between the temperature scale (ITS-90) and thermodynamic temperature or more recently determining the value for Boltzmann's constant for use in the forthcoming redefinition of the kelvin [3]. Despite efforts to develop Johnson noise thermometers for industrial applications where current technology does not provide adequate long-term stability (such is civil nuclear power generation [4]), there are currently no industrial thermometers based on Johnson noise.

The problem of immunity to external electromagnetic interference when measuring the small Johnson noise signal is surmountable by good design. The measurement of the Johnson noise in the presence of the electrical noise generated by the pre-amplifiers requires the use of correlation. However, long correlation times (hours or minutes depending on the uncertainty required) as used in historical systems, are incompatible with the requirements of industrial temperature measurement (a few seconds).

This paper describes the prior art in Johnson noise thermometry (section 2) and the origin of the unacceptably long measurement times caused by the need to switch between the device under test and a reference signal. It then describes a new measurement technique (section 3) that overcomes the need to switch and presents an analysis of the consequences on the measurement. A proof-of-principle prototype has been built to demonstrate this concept and results are presented (section 4) showing performance that is compatible with the requirements of industrial temperature measurements.

2. Prior Art Johnson Noise Thermometers

Johnson noise is the electrical noise generated by any resistor or dissipative impedance resulting from the thermal excitation of the charge carriers (usually electrons). It was first observed and reported by Johnson [5] and explained by Nyquist [6]. This thermal noise is white (equal power spectral density at all frequencies) and given by equation 1:

$$V_n = \sqrt{4\mathrm{k}TR\,\Delta f} \qquad \text{Equation 1}$$

where V_n= the root mean square (RMS) noise voltage, k = Boltzmann's constant, T= the thermodynamic temperature of the resistor, R= the resistance of the resistor and Δf = the measurement bandwidth.

Rearranging this equation, we get equation 2:

$$T = \frac{1}{4\mathrm{k}}\left(\frac{V_\mathrm{n}^2}{R}\,\frac{1}{\Delta f}\right) \qquad \text{Equation 2}$$

From equation 2 it is evident that if we connect a resistive sensor to a true RMS meter and measure the noise power (V_n^2/R) over a defined bandwidth (Δf) we can determine its true thermodynamic temperature. The required measurements include measuring the resistance of the sensor R in-situ, which is readily achieved. Importantly, the temperature measurement is unaffected by changes in any property of the sensor, thereby making the measurement technique independent of any calibration.

There are two very significant challenges with this apparently simple measurement technique; the very small signals involved and the difficulty in determining the measurement bandwidth.

2.1 Johnson Noise Signal Levels

The Johnson noise signals are extremely small and at the limits of measurement. For example, a typical historical Johnson noise thermometer would use a 100 Ω resistance and 100 kHz bandwidth, which generates a root mean square (RMS) noise signal at 20 °C of only 0.4 µV RMS (equation 1). To measure to an uncertainty of < 1 °C at 1000 °C (equivalent to 1 K at 1273.15 K i.e. 0.08 %) requires measuring the Johnson noise to an uncertainty of < 0.33 nV. Even the best low-noise amplifiers have an equivalent input noise of around 1 nV $/\sqrt{\mathrm{Hz}}$, giving an RMS noise over 100 kHz of 0.3 µV RMS. The noise from the amplifier is therefore comparable to the Johnson noise signal we need to measure.

The problem of the small signal can be resolved by using a signal correlator. This can be a time-domain correlator ([2], figure 4), or a frequency domain correlator as shown below.

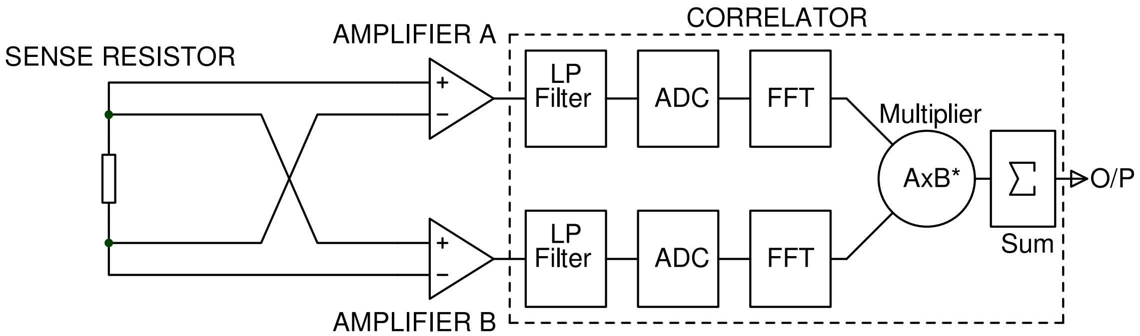

Figure 1: Correlator based Johnson noise measurement system.

The Johnson noise signal from the sense resistor is amplified simultaneously by two low-noise, high-gain amplifiers A and B. The output from each of these comprises an amplified version of the Johnson noise we wish to measure plus unwanted additional noise from the amplifier. Crucially, the additional voltage noise from the two amplifiers is uncorrelated, provided that the amplifiers and their power supplies are properly designed. The amplifier outputs are then filtered (to avoid aliasing), digitized by an analogue-to-digital converter (ADC), and transformed into the frequency domain using a fast Fourier transform (FFT). The data in the FFT bins associated with one amplifier channel is multiplied by the complex conjugate of the corresponding FFT bin from the other amplifier to determine the noise power for that frequency interval. The bins can then be summed over the required bandwidth to determine the RMS noise power. The correlator suppresses uncorrelated signals and selects only the signal common to both channels, which is the Johnson noise signal we wish to measure.

There is some correlated noise from the amplifiers caused by the input current noise associated with the amplifiers. This flows in the sense resistor where it generates a noise voltage that is common to both channels and therefore is not suppressed by correlation. However, by using a JFET (junction field-effect transistor) front-end to the amplifier this can be made negligible.

In the absence of current noise, the fractional uncertainty of the temperature determination is the fractional uncertainty of the noise power measurement and is given by equation 3 ([4], section 11.4.2):

$$\frac{u(T)}{T} = \frac{\sigma_{\overline{\mathrm{V_{o1}V_{o2}}}}}{E\left(\overline{\mathrm{V_{o1}V_{o2}}}\right)} = \sqrt{\frac{1}{\Delta t \Delta f_C}}\sqrt{1+\frac{1}{2}\left(\frac{\sigma_{\mathrm{V_{a1}}}^2+\sigma_{\mathrm{V_{a2}}}^2}{\sigma_{\mathrm{V_s}}^2}+\frac{\sigma_{\mathrm{V_{a1}}}^2\sigma_{\mathrm{V_{a2}}}^2}{\left(\sigma_{\mathrm{V_s}}^2\right)^2}\right)} \qquad \text{Equation 3}$$

where $\sigma_{\mathrm{v_{a1}}}$ and $\sigma_{\mathrm{v_{a2}}}$ are the equivalent RMS input voltage noise for the two amplifiers (taking into account the processing gain of the correlator), $\sigma_{\mathrm{v_s}}$ is the RMS Johnson noise we are measuring, $\sigma_{\overline{\mathrm{v_{o1}v_{o2}}}}$ is the standard deviation of the measurements of the noise power of expected value $E(\overline{\mathrm{v_{o1}v_{o2}}})$, Δt is the sample period and Δf_C the correlation bandwidth.

The unity term within the square root is the uncertainty arising from making an estimate of a random variable from a finite sample of the signal and corresponds to Rice's equation [2], [7]. It can also be derived from applying statistical mathematics to the behavior of the Gaussian Johnson noise signal [8]:

$$\frac{\sigma_V}{V} = \sqrt{\frac{1}{\Delta t \Delta f}} \qquad \text{Equation 4}$$

where σ_{V} is the standard deviation of measurements of the Gaussian variable of mean square (MS) value V made over a period Δt and bandwidth Δf.

This represents the ultimate uncertainty limit, with the remaining terms in equation 3 arising from the input voltage noise of the amplifiers used. It is clearly desirable to make $\sigma_{\mathrm{v_s}} \gg \sigma_{\mathrm{v_{a1}}}$ and $\sigma_{\mathrm{v_{a2}}}$ so as to approach this uncertainty limit. The technique described in this paper facilitates his by allowing the use of higher resistances that therefore generate more Johnson noise.

2.2 Bandwidth Determination

Prior-art systems switch between measuring the noise from the sensor resistance and a calibrated white noise source of known spectral power density. This source can simply be a reference resistor of known resistance at a known temperature ([9], figure 1). The input switch is used to alternate between measuring the Johnson noise from the sense resistor (V_{n} from R at temperature T over bandwidth Δf) and the reference resistor (V_0 from R_0 at temperature T_0 over bandwidth Δf_0). From [9]:

$$T = T_0 \left(\frac{V_{\mathrm{n}}}{V_0}\right)^2 \frac{R_0}{R} \qquad \text{Equation 5}$$

The problem is with the $\Delta f = \Delta f_0$ assumption leading to equation 5, since as we switch between the two measurements there will inevitably be a change in the frequency response of the measurement system. The dominant effect is the capacitance of the cables (C for the sensor resistor R, C_0 for reference resistor R_0) connecting the sensor and reference resistors to the input switch.

In order for the bandwidths of the two measurements to be the same, it is required that $RC = R_0C_0$. It is clearly possible to arrange for this criterion to be met for a particular measurement by trimming the circuit (for example, by adding capacitance to one set of cables to balance the system). However, if either resistor changes value with time or temperature there will be a mismatch in the effective measurement bandwidths, leading to measurement errors. This constraint therefore means that we would again be susceptible to changes in a property of the sensor, which is the very problem we are attempting to solve. There will also be a measurement uncertainly arising from how well these time constants can be matched in practice. The solution adopted in prior art systems was to ensure that both RC and R_0C_0 are $<< 1/\Delta f$ so that the measurement is made on the flat part of the system frequency response affected by these impedances. For practical systems involving cables, C and C_0 will be measured in tens of pF so this has previously limited the choice of parameters to resistances of around 100 Ω and bandwidths around 100 kHz. This limits the size of the Johnson noise signal to the previously stated low levels, which are comparable to the amplifier input noise.

A useful variant of the above technique involves replacing the reference resistor with a white pseudo-random noise (PRN) voltage source [1,8]. The digitally synthesized PRN signal is the sum a number of sinewaves (of equal amplitude) that are equally spaced in frequency and with random phase. This generates a Gaussian noise-like waveform that will repeat with a period of the reciprocal of the frequency spacing. The waveform can be stored as digital samples and then recreated by streaming the sampled waveform to a digital-to-analogue converter (DAC). In the most accurate Johnson noise thermometers used to investigate the alignment of temperature scales with thermodynamic temperature, the PRN signal may be generated using a Josephson junction array [1]. The “comb” of tones comprising the PRN calibration signal samples the frequency response of the measurement system.

3. New Johnson Noise Thermometer Measurement Topology

3.1 The Topology

Prior art Johnson noise thermometers were limited to using low value resistances and limited bandwidths for the reasons stated above. These limitations all result from the switching of the electronics between measuring the unknown Johnson noise signal and a known white noise reference signal. A new measurement topology was therefore devised to overcome the need to switch between the measurements, as shown in figure 2:

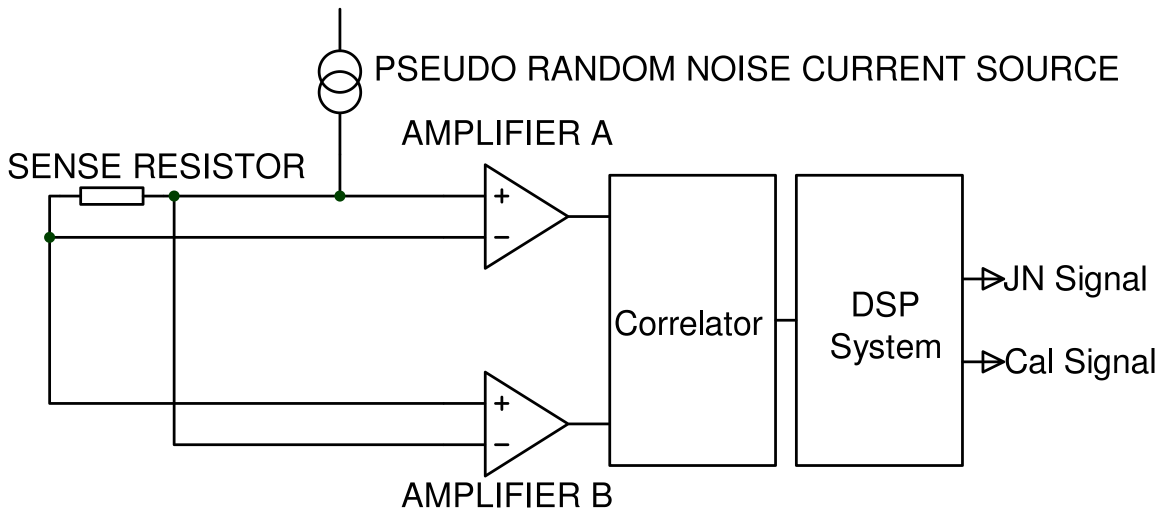

Figure 2: New Johnson noise thermometer topology.

The calibration signal (a PRN waveform) is injected as a current into the measurement circuit. This generates a calibration voltage signal in the sensor resistor, simultaneously with the Johnson noise signal. The calibration signal and the Johnson noise signal then experience the same frequency response as they pass through the connecting cables, amplifiers and correlator. The composite signal comprising the Johnson noise and calibration signal are then separated by a DSP system. The ratio of the Johnson noise and calibration signal can then be calculated in order to determine the absolute Johnson noise power and therefore the temperature of the sense resistor. This topology is tolerant of a “non-flat” frequency response since the two signals experience the same frequency response. As a result, it is possible to use a much wider bandwidth (1 MHz or more) when measuring the Johnson noise signal, including frequencies where there is significant attenuation arising from the connecting cables.

In our initial realization of this system, the PRN signal comprises tones (with randomized phase) at 152 Hz intervals between 10 kHz & 1.2 MHz. Lower frequency tones are used to determine the resistance of the sense resistor using Ohm's law (the amplitude of the tones in the current signal and the voltage gain of the electronics are determined by calibration of the electronics), since there is negligible attenuation by the cable capacitance at these frequencies.

The PRN current is generated using a 20 MS/s, 16-bit Keysight 33500B arbitrary function generator, which recreates the signal from a stored waveform. The voltage signal is converted to a current using a high value series resistor. The waveform at the output of the amplifiers is digitized using a 20 MS/s, dual-channel, 16-bit Agilent L4532A data acquisition unit, which was calibrated by programming it to create a range of sine waves that were then measured using an Agilent 34410A 6.5-digit digital multimeter (DMM).

The clock for the PRN current source and the digitizer in the correlator are common so that the calibration tones can be positioned at the exact center of each FFT bin. This means that there is no leakage of signal from the calibration tones into adjacent FFT bins as shown in figure 3:

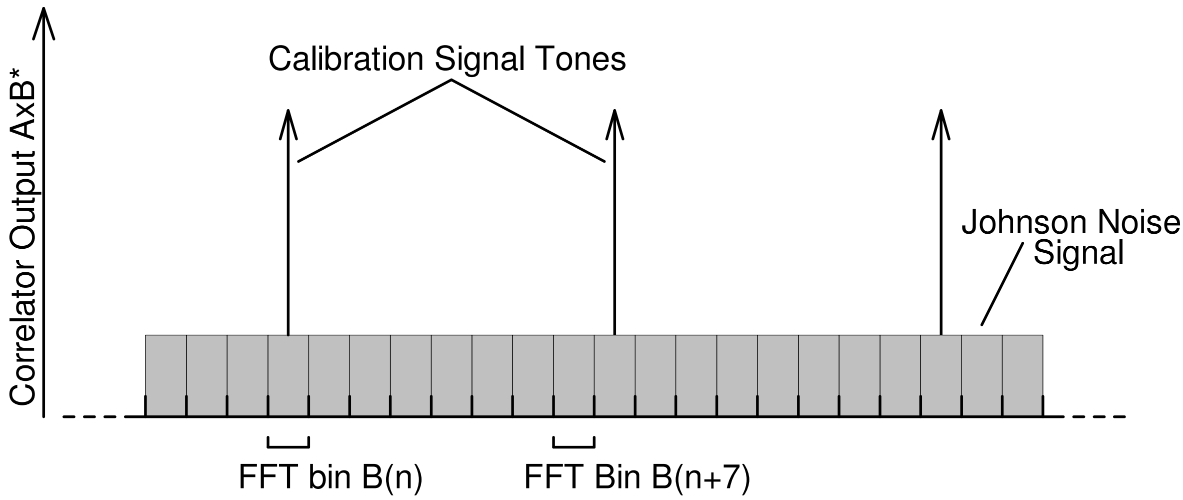

Figure 3: Correlator output from the new Johnson noise measurement system. The spacing of the calibration tones is 7 bins between the calibration tones. The illustration of the power of the calibration tones at the correlator output is schematic, in reality it is many times that of the Johnson noise. The calibration tones were 30 dB larger than the Johnson noise in the results quoted.

The Johnson noise power can be calculated by summing only the squared voltage in FFT bins that do not contain calibration tones. The calibration signal power can be determined by summing only those bins that do contain calibration tones and then subtracting the Johnson noise power just calculated. In this way, the composite signal can be completely separated into a Johnson noise power signal and a calibration power signal.

One of the problems with this concept is that the distributed resistance, inductance and capacitance of a transmission line delivering the calibration current signal to the sensor resistor would attenuate the higher frequency tones, leading to a measurement error. For this reason and practical reasons associated with making the connections between the electronics and the sensor resistor, the calibration current is instead injected at a point adjacent to one of the pre-amplifier inputs. Figure 4 shows this arrangement after folding out the circuit in figure 2:

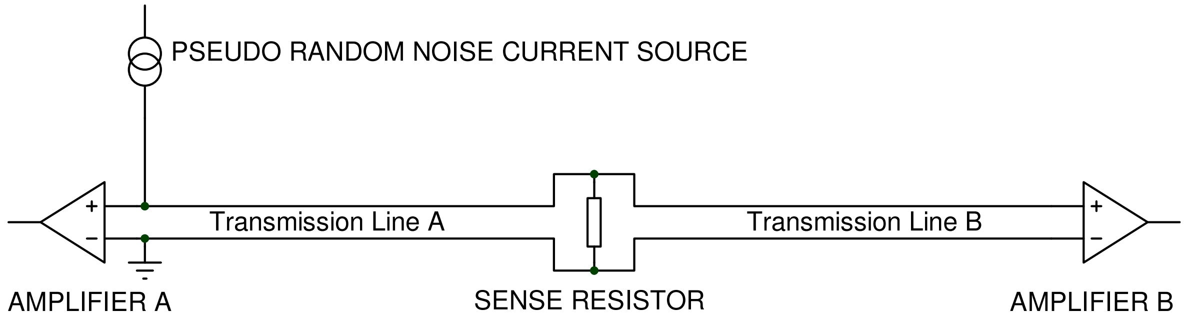

Figure 4: Transmission lines and current injection for the for new Johnson noise measurement.

A simplified model of the effect of the transmission lines can be achieved by proposing that the insertion of a single transmission line causes a fractional insertion loss factor x in the signal. Had the calibration current I been delivered directly to the sense resistor (the ideal position) without loss, the calibration signal reaching each amplifier input A and B would be reduced by a factor $(1-x)$ and the measured power from the calibration signal would have been:

$$P_C = I^2R(1-x)^2 \qquad \text{Equation 6}$$

For the situation shown in figure 4, the signal at A will be slightly higher than the ideal situation (having experienced no attenuation) and the signal at B would be slightly lower, having experienced the attenuation due to two transmission lines. The measured calibration signal power is then:

$$P'_{\mathrm{C}} = \frac{\{V'_{\mathrm{A}}\}\{V'_{\mathrm{B}}\}}{R} = \frac{\{IR\}\{IR(1-x)^2\}}{R} = I^2R(1-x)^2 \qquad \text{Equation 7}$$

Equation 7 implies that the transmission lines contribute no error if the calibration current is injected at the input to one of the amplifiers. However, the model is oversimplified and specifically does not take into account the effect of the sense resistor at the mid-point of the transmission line network. By treating the cables as transmission lines with distributed resistance, inductance and capacitances, the input network was modelled numerically and the effect of the transmission lines explored for various system parameters and optimized using a Monte Carlo method. As might be expected from the above description, the errors reduce as the value of the sense resistor is increased (since this removes the asymmetry caused by the sensor resistance). Figure 5 show the effect of increasing the sensor resistance for a cable with total series resistance of 1 Ω, transmission lines of capacitance 0-100 pF and bandwidths of 0-1 MHz:

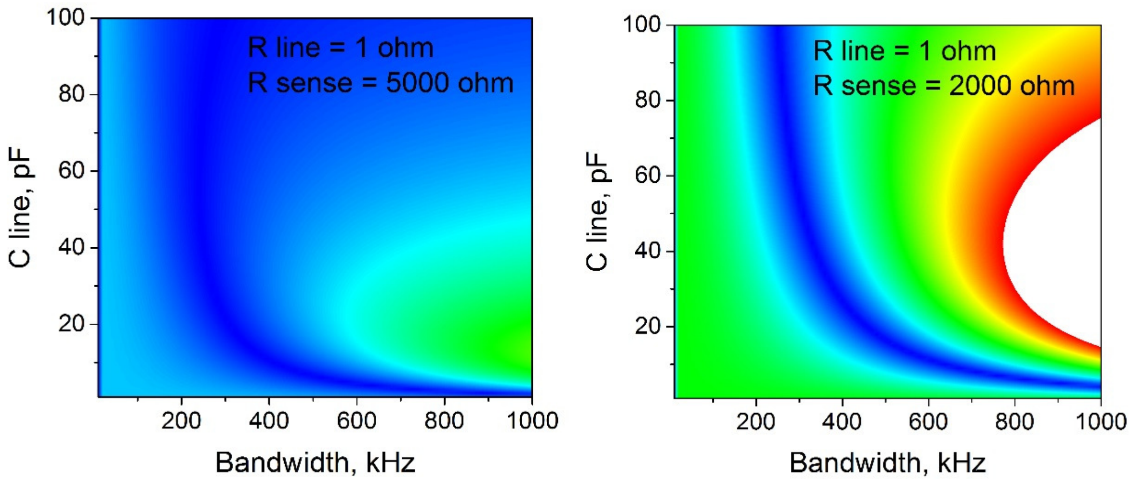

Figure 5: Effect of sense resistance on measurement. Color corresponds to the magnitude of the error, ranging from 0% (blue) to 0.1% (red).

The dark blue areas are those of low error and the colors increase with error up to the high error areas shown in red. These graphs clearly show the benefit of using a higher sense resistance, with the extensive blue areas shown for the 5 kΩ sense resistor on the left compared with the more limited blue area for a 2 kΩ sense resistor shown on the right. The errors arising from the injection of the current at the input to one of the amplifiers gets smaller as the sense resistor increases in value. Increasing the sensor resistor value also increases the magnitude of the Johnson noise signal (equation 1). We are also able to use a much higher bandwidth as we are no longer limited to operating on the flat part of the input circuit's frequency response.

The high value resistor and consequent increased Johnson noise signal means that the measurement uncertainty closely approaches the fundamental limit described by Rice's equation. The ability to use a higher bandwidth and the lack of switching mean that the measurement time for a given uncertainty (determined by this fundamental limit) is minimized. In our implementation this reduces the measurement time by a factor of 20 (1 MHz c.f. 100 kHz bandwidth and x2 for no switching).

3.2 Measuring Temperature With The New Topology

A measurement of temperature requires measuring the Johnson noise power and the resistance of the sensor (equation 2). The measurement of the noise power requires the current source amplitudes to be well known. The measurement of the resistance additionally requires that the gains of the measurement electronics be known so that a traceable measurement can be made. For this reason, only tones at lower frequencies (where the transmission line capacitance has negligible effect on the signals) are used to determine the sensor resistance. The gains of the electronics at the higher frequencies (included in the noise power measurement) do not need to be known since the PRN current calibration signal effectively characterizes the system over the measurement bandwidth.

In the present realization of the system, the current source is replaced by its Thevenin equivalent, specifically a PRN voltage source and a high value resistor. The resistor, however, contributes Johnson noise of its own, which must be subtracted from the measurements. The Johnson noise from the sense resistor (V from R at temperature T) and the Johnson noise from the calibration signal feed-in resistor (V_{fi} from R_{fi} at temperature T_{fi}) are modelled as voltage sources in series with noiseless resistors.

3.2.1 Measuring Sensor Resistance

The calibration current comprises a "comb" of sinewaves, each of RMS voltage V_{CN} (with random phase) which are positioned at the center of an FFT bin B_N and combine with R_{fi} to generate the calibration current. The calibration current must be known so that each FFT bin has units of V^2.

Consider a bin N that contains one of the tones, the value at the output of the correlator (being the product of the bin for channel A multiplied by the complex conjugate of channel B) will be:

$$B_N{}^2 = V_{CN}{}^2 \left(\frac{R}{\mathrm{R_{fi}}+R}\right)^2 + 4\mathrm{k}T_{\mathrm{fi}}R_{\mathrm{fi}}\delta f \left(\frac{R}{R_{\mathrm{fi}}+R}\right)^2 + 4\mathrm{k}TR\delta f \left(\frac{R_{\mathrm{fi}}}{R_{\mathrm{fi}}+R}\right)^2$$

$$\therefore B_N{}^2 = V_{CN}{}^2 \left(\frac{R}{R_{\mathrm{fi}}+R}\right)^2 + 4\mathrm{k}\frac{(T_{\mathrm{fi}}R+TR_{\mathrm{fi}})R_{\mathrm{fi}}\,R}{(R_{\mathrm{fi}}+R)^2}\delta f \qquad \text{Equation 8}$$

where δf = the bandwidth of the FFT bins.

In order to use equation 8 to determine the sensor resistance (R), we would have to know the temperature of the sensor resistance (T). The derived value of R will eventually be used to determine T so it would be possible to use an iterative approach to determining both R and T. However, a simpler approach is to use adjacent FFT bins to measure the Johnson noise from both the sensor resistance and the feed-in resistor. If the adjacent FFT bin does not contain a calibration tone, then:

$$B_{N+1}{}^2 = 4\mathrm{k}\frac{(T_{\mathrm{fi}}R+TR_{\mathrm{fi}})R_{\mathrm{fi}}\,R}{(R_{\mathrm{fi}}+R)^2}\delta f \qquad \text{Equation 9}$$

By substitution into equation 8, we can determine the sensor's resistance from these two measurements as follows:

$$B_N{}^2 = V_{CN}{}^2 \left(\frac{R}{R_{\mathrm{fi}}+R}\right)^2 + B_{N+1}{}^2$$

$$\therefore R = R_{\mathrm{fi}}\frac{\sqrt{B_N{}^2 - B_{N+1}{}^2}}{V_{CN} - \sqrt{B_N{}^2 - B_{N+1}{}^2}} \qquad \text{Equation 10}$$

In practice, we use the average of several (2*M*) FFT bins either side of bin N to decrease the uncertainty on B_{N+1}.

3.2.2 Uncertainty in Measuring Sensor Resistance

The in-situ measurement of the sensor resistance carries the full accuracy burden of the temperature measurement. It is therefore vital to understand the sources of uncertainty and to ensure that the resistance is determined to an adequately small uncertainty. The $4\mathrm{k}\frac{(T_{\mathrm{fi}}R+TR_{\mathrm{fi}})R_{\mathrm{fi}}\,R}{(R_{\mathrm{fi}}+R)^2}\delta f$ terms in equations 8 and 9 (for B_N and B_{N+1}) are Johnson noise signals. The values measured by the system over a finite period (Δt) will be estimates of the Johnson noise and will therefore be subject to uncertainties. These uncertainties are the dominant uncertainty in our measurement of resistance. From equations 4 and 8, the uncertainty (coverage factor *k*=1) associated with B_N and B_{N+1} is:

$$u_{\mathrm{B}X} = \frac{1}{\sqrt{\Delta t\,\Delta f}}\sqrt{4\mathrm{k}\frac{(T_{\mathrm{fi}}R+TR_{\mathrm{fi}})R_{\mathrm{fi}}\,R}{(R_{\mathrm{fi}}+R)^2}\delta f} \qquad \text{Equation 11}$$

In our system, $V_{CN} \gg \sqrt{{B_N}^2 - {B_{N+1}}^2}$, so from equation 10:

$$R \approx R_{\text{fi}} \frac{\sqrt{{B_N}^2 - {B_{N+1}}^2}}{V_{CN}} \qquad \text{Equation 12}$$

It is evident from equation 12 that the fractional uncertainty of the resistance measurement (σ_R/R) equals the fractional uncertainty of $\sqrt{{B_N}^2 - {B_{N+1}}^2}$. Since the Johnson noise term is much smaller than the calibration tone in B_N and $\Delta f \approx \delta f$, equations 8 and 11 show this is given by:

$$\frac{\sigma_R}{R} \approx \sqrt{2} \sqrt{\frac{4\text{k}(T_{\text{fi}}R + TR_{\text{fi}})R_{\text{fi}}\,R}{(R_{\text{fi}}+R)^2 \Delta t}} \;\frac{(R_{\text{fi}}+R)}{R\,V_{CN}} \qquad \text{Equation 13}$$

Since $R_{\text{fi}} >> R$ and T_{fi} will generally be $< T$, this reduces to:

$$\frac{\sigma_R}{R} \approx \sqrt{\frac{8\text{k}TR}{\Delta t}}\;\frac{R_{\text{fi}}}{R\,V_{CN}} \qquad \text{Equation 14}$$

For a system operating with $T = 1000$ K, $R = 5$ kΩ, $\Delta t = 65.5$ s, $R_C/R = 100$ and $V_{CN} = 5$ mV, the fractional uncertainty (k=1) would be ~ 0.006 % (i.e. insignificant for industrial measurements). Our measurement system samples the signal over a 6.55 s period. Since the resistance of the sensor does not change rapidly, ten successive measurements for each bin can be used to form the 65.5 s sample period required in the above evaluation of equation 14.

3.2.3 Measuring Temperature

The temperature is derived by calculating the ratio X of the signal (voltage squared) in a tone bin B_N to $2M$ adjacent bins not containing a tone. These should be spaced equally either side of the tone bin. The ratio X is therefore:

$$X = \frac{B_N}{\sum_{n=N-M}^{n=N-1} B_n + \sum_{n=N+1}^{n=N+M} B_n} = \frac{{V_{\text{CN}}}^2\left(\frac{R}{R_{\text{fi}}+R}\right)^2 + 4\text{k}\frac{(T_{\text{fi}}R + TR_{\text{fi}})R_{\text{fi}}\,R}{(R_{\text{fi}}+R)^2}\delta f}{2M4\text{k}\frac{(T_{\text{fi}}R + TR_{\text{fi}})R_{\text{fi}}\,R}{(R_{\text{fi}}+R)^2}\delta f}$$

$$\therefore T = \frac{({V_{\text{CN}}}^2 - (2MX-1)4\text{k}T_{\text{fi}}R_{\text{fi}}\delta f)R}{(2MX-1)4\text{k}{R_{\text{fi}}}^2\delta f} \qquad \text{Equation 15}$$

To determine the effect of uncertainty in X arising from the fact that the data is derived from a finite sample of the noise, we need to differentiate equation 15:

$$\frac{dT}{dX} = \frac{-2M4kT_{\text{fi}}R_{\text{fi}}\delta f R}{(2MX-1)4k{R_{\text{fi}}}^2\delta f} - \frac{({V_{\text{CN}}}^2 - (2MX-1)4\text{k}T_{\text{fi}}R_{\text{fi}}\delta f)R}{\{(2MX-1)4\text{k}{R_{\text{fi}}}^2\delta f\}^2}2M4\text{k}{R_{\text{fi}}}^2\delta f$$

$$\therefore \frac{dT}{dX} = \frac{-2MR{V_{\text{CN}}}^2}{(2MX-1)^2 4\text{k}{R_{\text{fi}}}^2\delta f}$$

The relationship between the fractional uncertainty in X and T is given by:

$$\frac{\delta T}{T} = \frac{\delta X}{X} \frac{-2MRV_{CN}^{2}}{(2MX-1)^{2}4\mathrm{k}R_{\mathrm{fi}}^{2}\delta f} \frac{(2MX-1)4\mathrm{k}R_{\mathrm{fi}}^{2}\delta f}{\left(V_{CN}^{2}-(2MX-1)4\mathrm{k}T_{\mathrm{fi}}R_{\mathrm{fi}}\delta f\right)R} X$$

$$\therefore \frac{\delta T}{T} = \frac{\delta X}{X} \frac{-2MXV_{CN}^{2}}{(2MX-1)\left(V_{CN}^{2}-(2MX-1)4\mathrm{k}T_{\mathrm{fi}}R_{\mathrm{fi}}\delta f\right)}$$

Provided $V_{\mathrm{CN}}^{2} \gg (2MX-1)4\mathrm{k}T_{\mathrm{fi}}R_{\mathrm{fi}}\delta f$ (i.e. the calibration signal is substantially above the Johnson noise from the feed-in resistor) and $2MX \gg 1$ (which is true in our system where the calibration signal exceeds the Johnson noise signal in the bins), this reduces to:

$$\therefore \frac{\delta T}{T} \approx \frac{\delta X}{X} \frac{-2MX}{(2MX-1)} \approx \frac{\delta X}{X} \approx \frac{1}{\sqrt{\Delta t \, \Delta f}} \qquad \text{Equation 16}$$

With the above provisos, the uncertainty in the determination of temperature is dominated by the uncertainty associated with measuring noise power from a limited time sample, all other sources, such as the calibration of the measurement electronics, are negligible by comparison.

There are a large number of bins available from the correlator output and there are two ways of using the information in these bins. We could calculate the ratio Xassociated with each calibration tone, then average, then apply equation 15. Alternatively, we could sum the calibration tones and the non-calibration tones and calculate a global ratio, then apply equation 15. We used the first approach since in our system the Johnson noise is well above the amplifier input noise and the uncertainty is therefore primarily limited by the uncertainty associated with the finite measurement interval of the noise sample. This means that the fractional uncertainty from higher frequencies bins (where the Johnson noise signal is attenuated significantly by the cable capacitance) will be substantially the same as for the lower frequency bins where the signal is higher. This approach decreases the response time significantly with no corresponding degradation in the fractional uncertainty. The non-linear nature of division in equation 15 does mean that there is a consequent bias caused by the variance in the FFT data [8], this is acceptably small in the present realization but will be investigated and eliminated in future research.

If we include the uncertainty arising from the resistance measurement, then for a measurement using Pbins to measure the sensor's resistance and Qbins to measure the Johnson noise, the fractional uncertainty in the temperature measurement (95 % confidence level) will be:

$$\frac{\delta T}{T} \approx 2\sqrt{\frac{8kTR}{P\,\delta f}\left(\frac{R_C}{R\,V_{CN}}\right)^{2} + \frac{1}{Q\,\delta t\,\delta f}} \qquad \text{Equation 17}$$

Entering the parameters for our initial realization of the system indicates that the fractional uncertainty for a measurement of Taround ambient temperatures should be 0.13%. This reduces to 0.08% if $P$is increased so as to make the first term insignificant. The drawback to this approach is that the immunity of the measurement to changes in the resistance of the sensor is reduced.

4. Results

Initial tests were conducted to ensure that the measured spectra were clean with no evidence of contamination from external noise sources. These early results showed contaminated spectra and highlighted the sensitivity of the system to external interference. The system was then modified to use a full tri-tri-axial connection at which point the spectra became consistently "clean".

Measurements (arbitrary power units) were then made of the Johnson noise from a 5 kΩ foil resistor at temperatures from -20 °C to 120 °C. The results showed a linear dependence on temperature (as expected) and the extrapolation intercepted the zero power axis at -271.79 ˚C (1.36 K) as shown in figure 6:

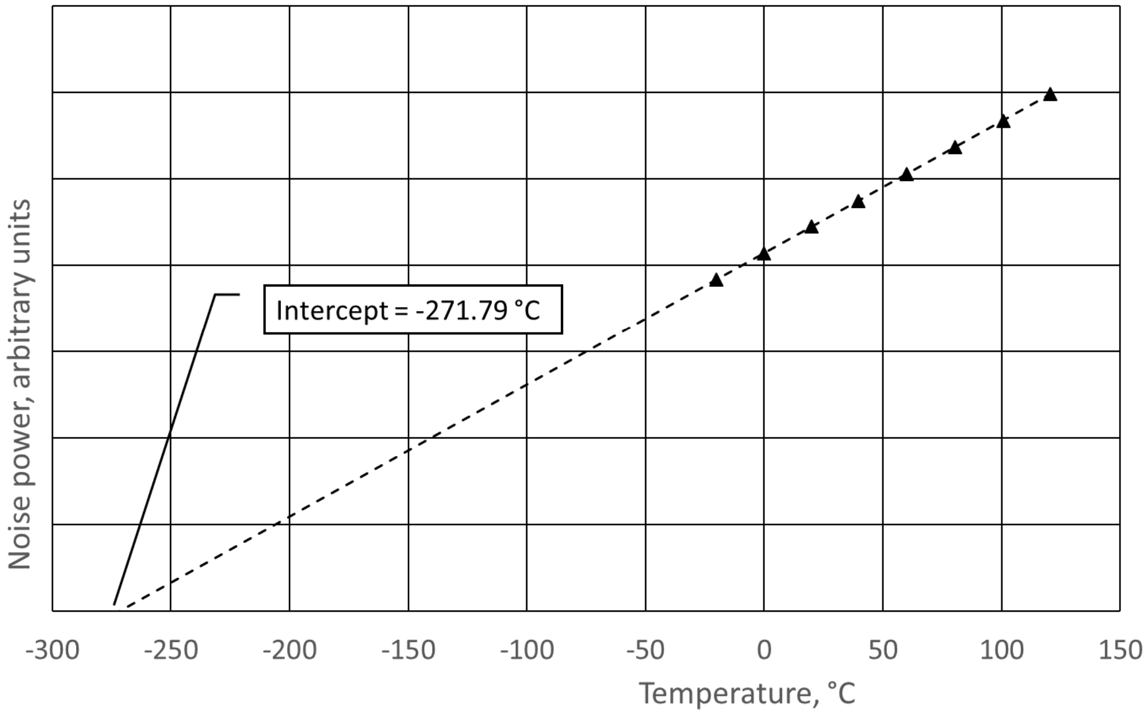

Figure 6: Measurement of noise power from a 5 kΩ resistor between -20 °C and +120 °C.

The system was extended to include the calibration tones allowing the absolute power and therefore the temperature of the sensor resistor to be determined. The temperature of a 5 kΩ foil resistor was then measured in an oil bath maintained at 20.055± 0.01 ˚C. A series of 100 readings were taken over a 3 day period as shown in figure 7:

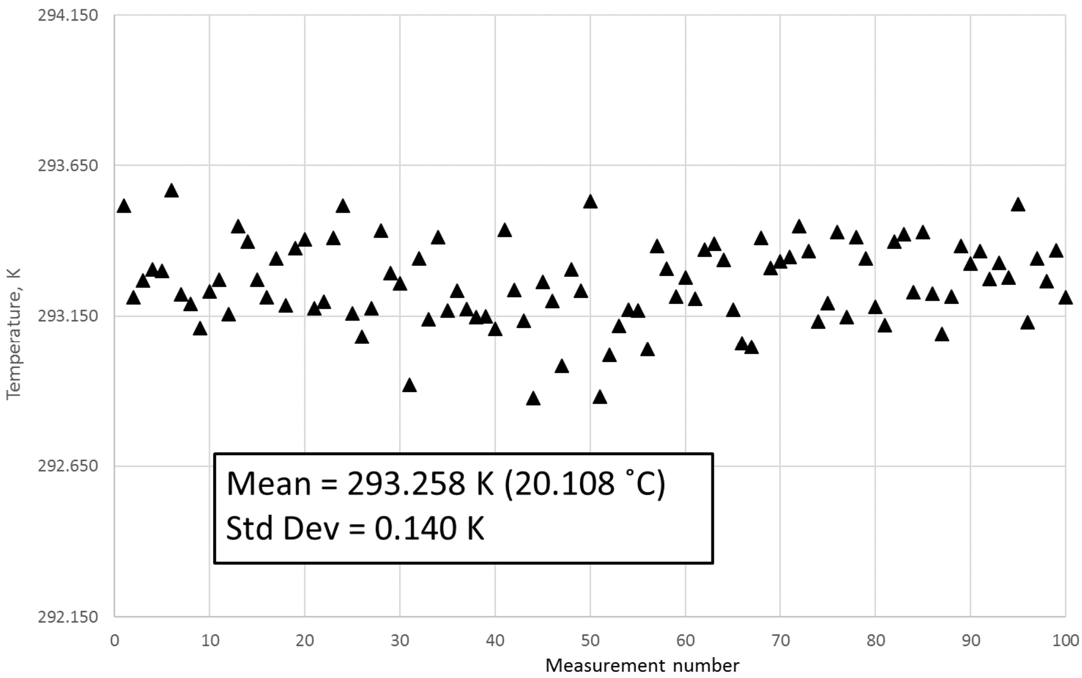

Figure 7: 100 measurements of temperature made over three days

The standard deviation of the temperature determined using the Johnson noise measurements was found to be 0.14 K, which is within a factor of two of the value expected from the above analysis.

5. Conclusions

A new technique for realizing a Johnson noise thermometer has been developed that allows the use of higher resistance values so as to minimize the effect of pre-amplifier noise and avoid switching. The technique allows the Johnson noise, calibration signal (used to characterize the response of the measurement system) and sensor resistance to be determine simultaneously (from the same sampled data set) so that it can operate at higher bandwidths (beyond the flat part of the system's frequency response). As a result of these developments, a Johnson noise thermometer can be realized whose uncertainty approaches the theoretical limit set by Rice's equation.

The new Johnson noise technique is capable of making measurements with uncertainties and response times that would meet the needs of many industrial applications. Specifically, it has been possible to make temperature measurements to a fractional uncertainty of ~0.1 % (95 % confidence) within 7 s (0.28 K at 293 K).

The technique can benefit from further refinement. In particular the authors intend to characterize the errors in the technique with the objective of providing first order correction of such errors. By also using longer measurement times, this will allow a further reduction in measurement uncertainty. The authors intend to develop the technology further with the aim of producing commercial, drift-free thermometers based on Johnson noise over the next few years.

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